MATILI TECHNICAL TRAINING INSTITUTE

2023

VOLUMES

Earthwork and Mass Diagrams

IWORDS ACADEMY

School for the smarties

LECTURES BY GATHIGIRA STEPHEN

www.youtube.com/@IwordsShotsEmpire

Terrain Effects on Route Location

- Earthwork is costly
- Attempt to minimize amount of earthwork necessary
  - Set grade line as close as possible to natural ground level
  - Set grade line so there is a balance between excavated volume and volume of embankment

http://www.agtek.com/highway.htm

Earthwork Analysis

- Take average cross-sections along the alignment (typically 50 feet)
- Plot natural ground level and proposed grade profile and indicate areas of cut and fill
- Calculate volume of earthwork between cross-sections

Average End Area Method

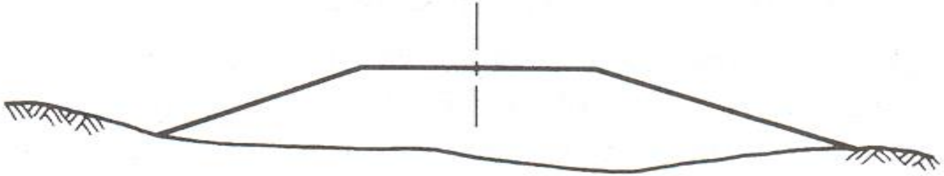
- Assumes volume between two consecutive cross sections is the average of their areas multiplied by the distance between them

$V = L(A_1 + A_2) \div 54$

V = volume (yd^3)

A_1 and A_2 = end areas of cross-sections 1 & 2 (ft^2)

L = distance between cross-sections (feet)



(a) Fill

(b) Cut

(c) Fill and cut

Source: Garber and Hoel, 2002

Shrinkage

- Material volume increases during excavation
- Decreases during compaction
- Varies with soil type and depth of fill

Swell

- Excavated rock used in embankment occupies more space
- May amount to 30% or more

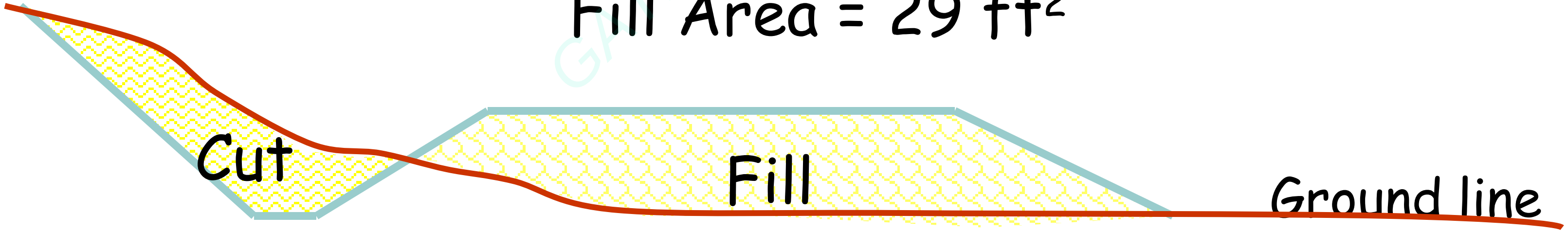
Computing Volume (Example)

Shrinkage = 10%, L = 100 ft

Station 1:

Cut Area = 6 ft^2

Fill Area = 29 ft^2



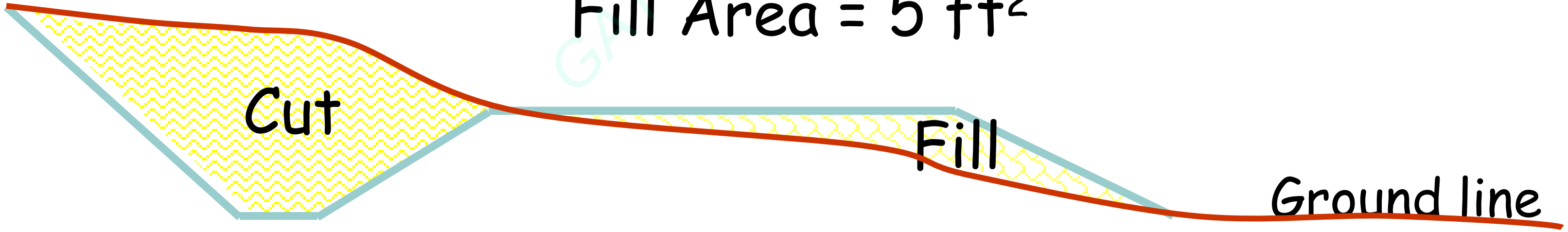
Computing Volume (Example)

Shrinkage = 10%

Station 2:

Cut Area = 29 ft^2

Fill Area = 5 ft^2



$$V_{cut} = \frac{L\,(A_{1cut} + A_{2cut})}{54} = \frac{100\ ft\ (6\ ft^2 + 29\ ft^2)}{54} = 64.8\ yd^3$$

$$V_{fill} = \frac{L\,(A_{1fill} + A_{2fill})}{54} = \frac{100\ ft\ (29\ ft^2 + 5\ ft^2)}{54} = 63.0\ yd^3$$

Fill for shrinkage = 63.0 * 0.1 = 6.3 yd^3

Total fill = 63.0 ft^3 + 6.3 ft^3 = 69.3 yd^3

Total cut and fill between stations 1 and 2 = 69.3 yd^3 fill – 64.8 yd^3 cut = **4.5 yd^3 fill**

*note: no allowance made for expansion

Station	End Area		Volume				Net Volume	
	Total Cut	Total Fill	Total Cut	Total Fill	Shrinkage 10%	Adjusted Fill	Fill	Cut
1	6.0	29.0						
2	29.0	5.0	64.8	63.0	6.3	69.3	−4.5	

Mass Diagram

- Series of lines that shows ***net*** accumulation of cut or fill between any 2 stations
- Ordinate is the net accumulation of volume from an arbitrary starting point
- First station is the starting point

Estimating End Area

Station 1:

Estimating End Area

Station 1: Fill Area = ∑Shapes

Calculate Mass Diagram Assuming Shrinkage = 25%

Station	End Area (ft^2)		Vol	Vol	Adjusted	Mass Diagram
	Cut	Fill	cut (yd)	Fill (yd)	Fill (yd)	Ordinate
0	40	0				
1	140	20				
2	160	25				
3	60	116				

Calculate Mass Diagram Assuming Shrinkage = 25%

Station	End Area (ft^2) Cut	Fill	Vol cut (yd)	Vol Fill (yd)	Adjusted Fill (yd)	Mass Diagram Ordinate
0	40	0				
			333.3	37.0		
1	140	20				
2	160	25				
3	60	116				

$$Volume_{cut} = \frac{100\ ft\ (40\ ft^2 + 140\ ft^2)}{54} = 333.3\ yd^3\ cut$$

$$Volume_{fill} = \frac{100\ ft\ (20\ ft^2 + 0\ ft^2)}{54} = 37.0\ yd^3\ fill$$

Calculate Mass Diagram Assuming Shrinkage = 25%

Station	End Area (ft^2)		Vol	Vol	Adjusted	Mass Diagram
	Cut	Fill	cut (yd)	Fill (yd)	Fill (yd)	Ordinate
0	40	0				
			333.3	37.0	46.3	
1	140	20				
2	160	25				
3	60	116				

$Volume_{fill}$ = adjusted for shrinkage = 37.0 yd * 1.25 = 46.3 yd^3

Calculate Mass Diagram Assuming Shrinkage = 25%

Station	End Area (ft^2)		Vol	Vol	Adjusted	Mass Diagram
	Cut	Fill	cut (yd)	Fill (yd)	Fill (yd)	Ordinate
0	40	0				
			333.3	37.0	46.3	287.0
1	140	20				
2	160	25				
3	60	116				

Total cut = 333.3 yd^3 - 46.3 yd^3 = 287.0 yd^3

Calculate Mass Diagram Assuming Shrinkage = 25%

Station	End Area (ft^2)		Vol	Vol	Adjusted	Mass Diagram
	Cut	Fill	cut (yd)	Fill (yd)	Fill (yd)	Ordinate
0	40	0				
			333.3	37.0	46.3	
1	140	20				287.0
			555.6	83.3	104.2	
2	160	25				
3	60	116				

$$Volume_{cut} = \frac{100\ ft\ (140\ ft^2 + 160\ ft^2)}{54} = \underline{555.6\ yd^3\ cut}$$

$$Volume_{fill} = \frac{100\ ft\ (20\ ft^2 + 25\ ft^2)}{54} = \underline{83.3\ yd^3\ fill}$$

$$Volume_{fill} = \text{adjusted for shrinkage} = 83.3\ yd * 1.25 = \underline{104.2\ yd^3}$$

$$\text{Total cut 1 to 2} = 555.6\ yd^3 - 104.2\ yd^3 = \underline{451.4\ yd^3}$$

Calculate Mass Diagram Assuming Shrinkage = 25%

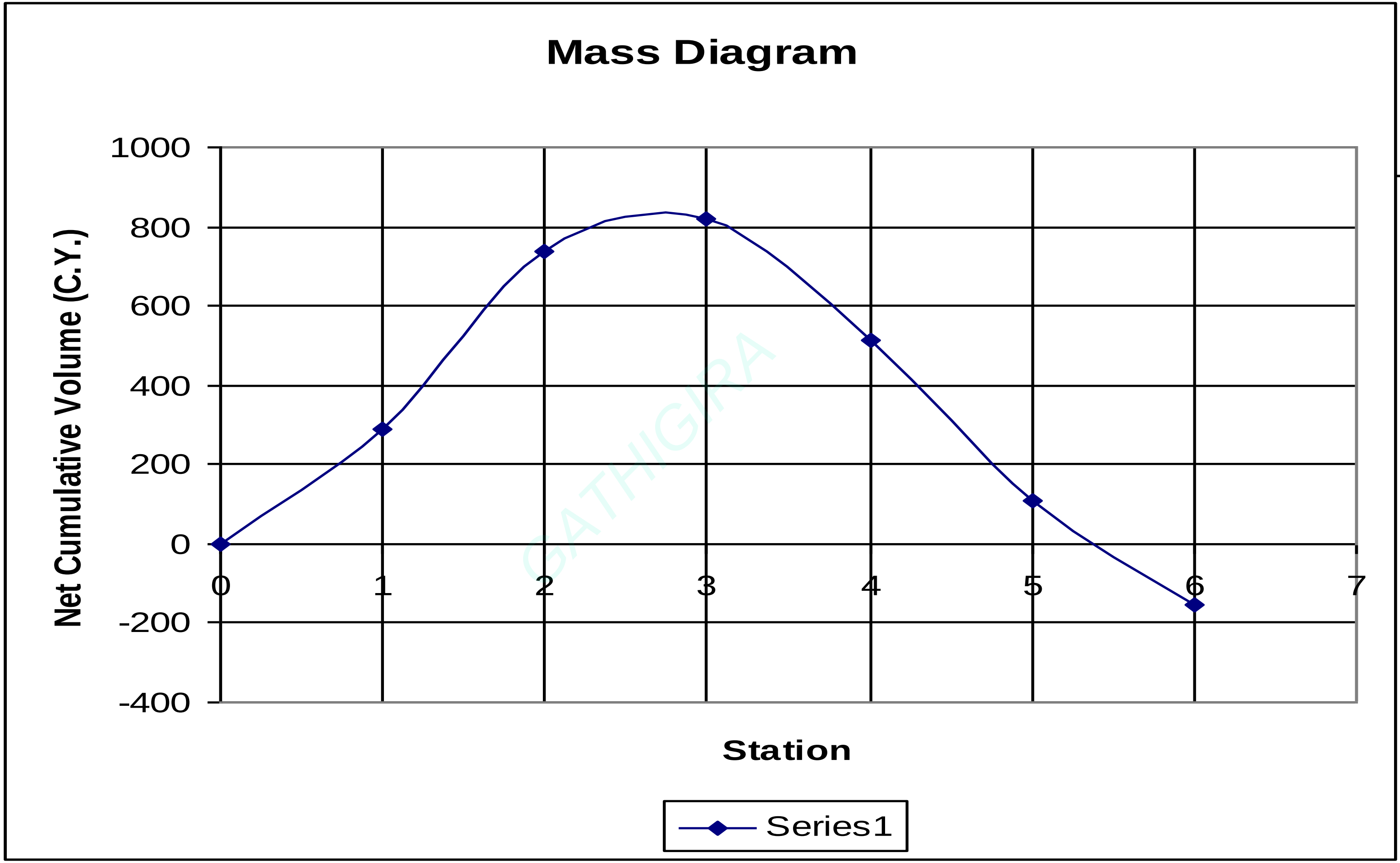
Station	End Area (ft^2)		Vol	Vol	Adjusted	Mass Diagram
	Cut	Fill	cut (yd)	Fill (yd)	Fill (yd)	Ordinate
0	40	0				
			333.3	37.0	46.3	
1	140	20				287.0
			555.6	83.3	104.2	
2	160	25				738.4
3	60	116				

Total cut = 451.4 yd^3 + 287 = 738.4 yd^3

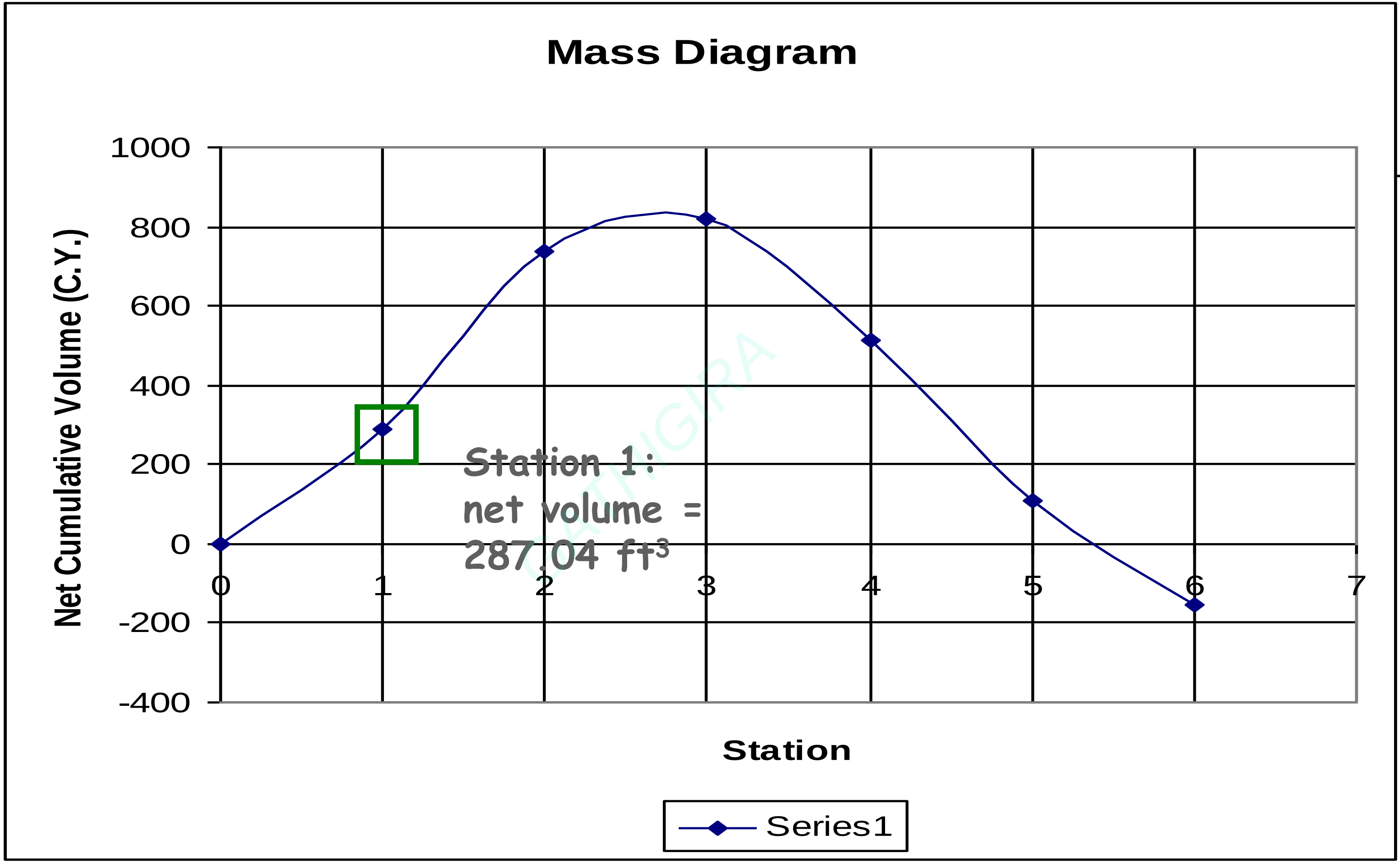
Calculate Mass Diagram Assuming Shrinkage = 25%

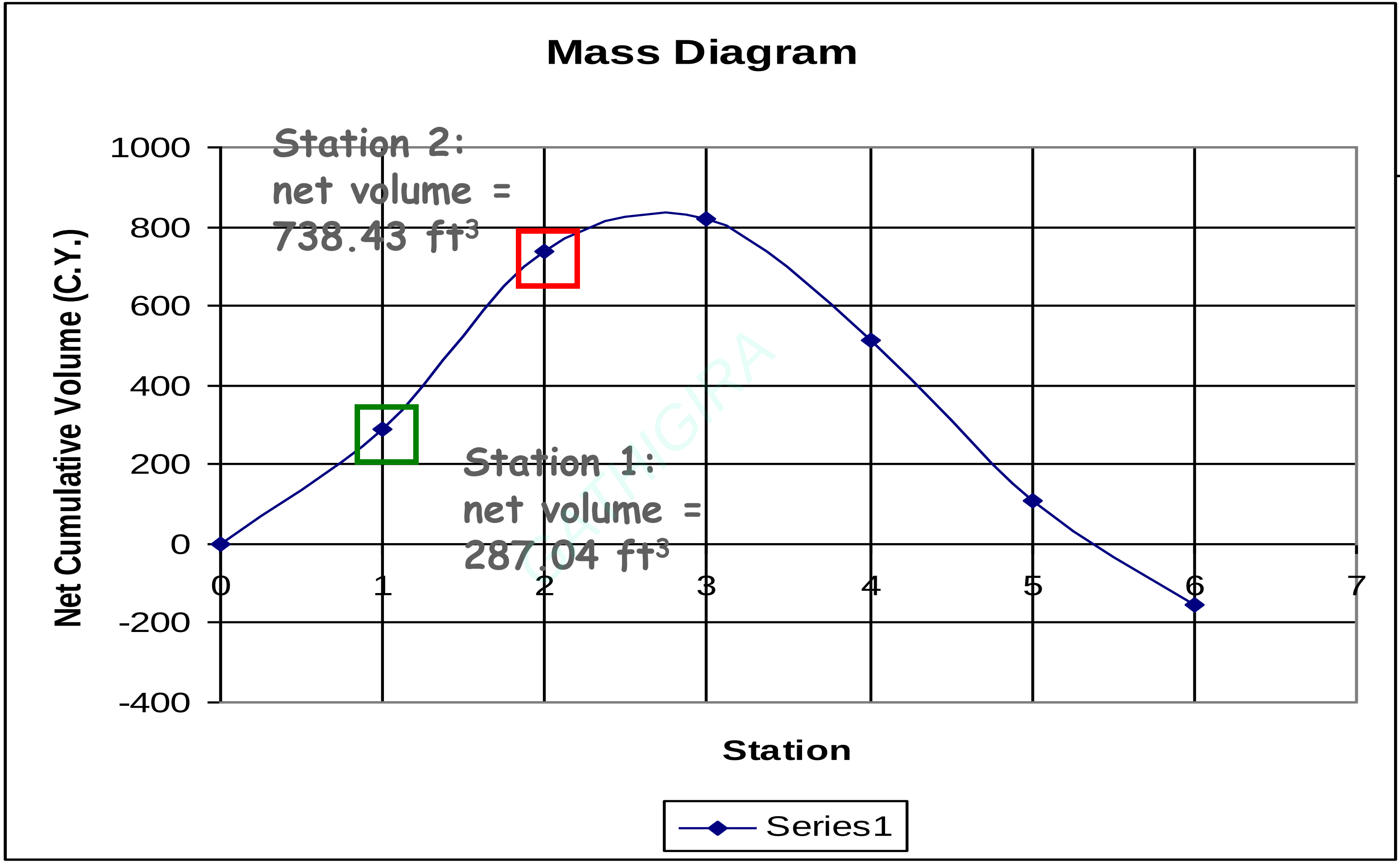
Station	End Area (ft^2)		Vol	Vol	Adjusted	Mass Diagram
	Cut	Fill	cut (yd)	Fill (yd)	Fill (yd)	Ordinate
0	40	0				
			333.3	37.0	46.3	
1	140	20				287.0
			555.6	83.3	104.2	
2	160	25				738.4
			407.4	261.1	326.4	
3	60	116				819.4

Final Station

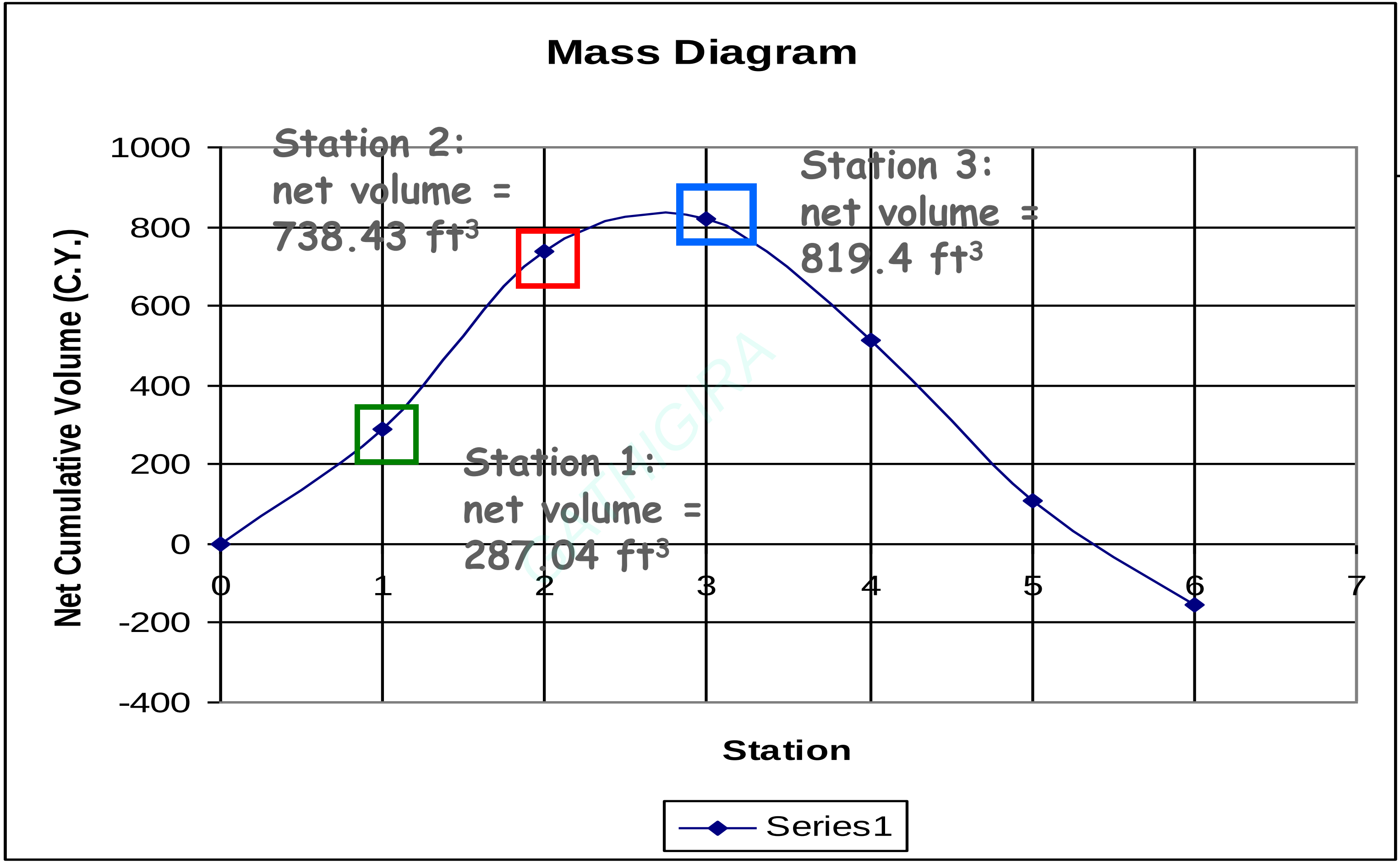


Mass Diagram
1000
800
600
400
200
0
-200
-400
Net Cumulative Volume (C.Y.)
0
1
2
3
4
5
6
7
Station
Series1
GATHIGIRA

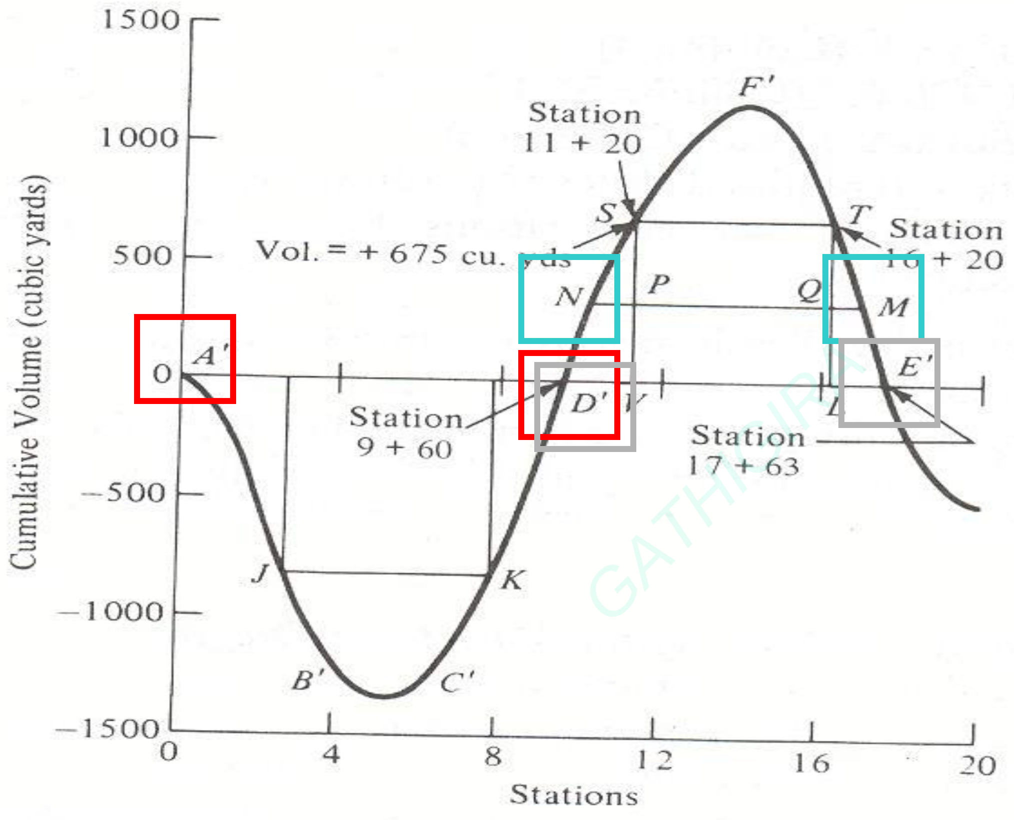

Mass Diagram
Net Cumulative Volume (C.Y.)
1000
800
600
400
200
0
-200
-400
0
1
2
3
4
5
6
7
Station
Station 1:
net volume =
287.04 ft3
Series1



Mass Diagram
Station 2:
net volume =
738.43 ft³
Station 1:
net volume =
287.04 ft³
Net Cumulative Volume (C.Y.)
1000
800
600
400
200
0
-200
-400
0
1
2
3
4
5
6
7
Station
Series1



Mass Diagram
Station 2:
net volume =
738.43 ft³
Station 3:
net volume =
819.4 ft³
Station 1:
net volume =
287.04 ft³
1000
800
600
400
200
0
-200
-400
0
1
2
3
4
5
6
7
Net Cumulative Volume (C.Y.)
Station
Series1



Mass Haul Diagram for Computation Shown in Table 15.1

Balance point: balance ~~of cut and fill~~

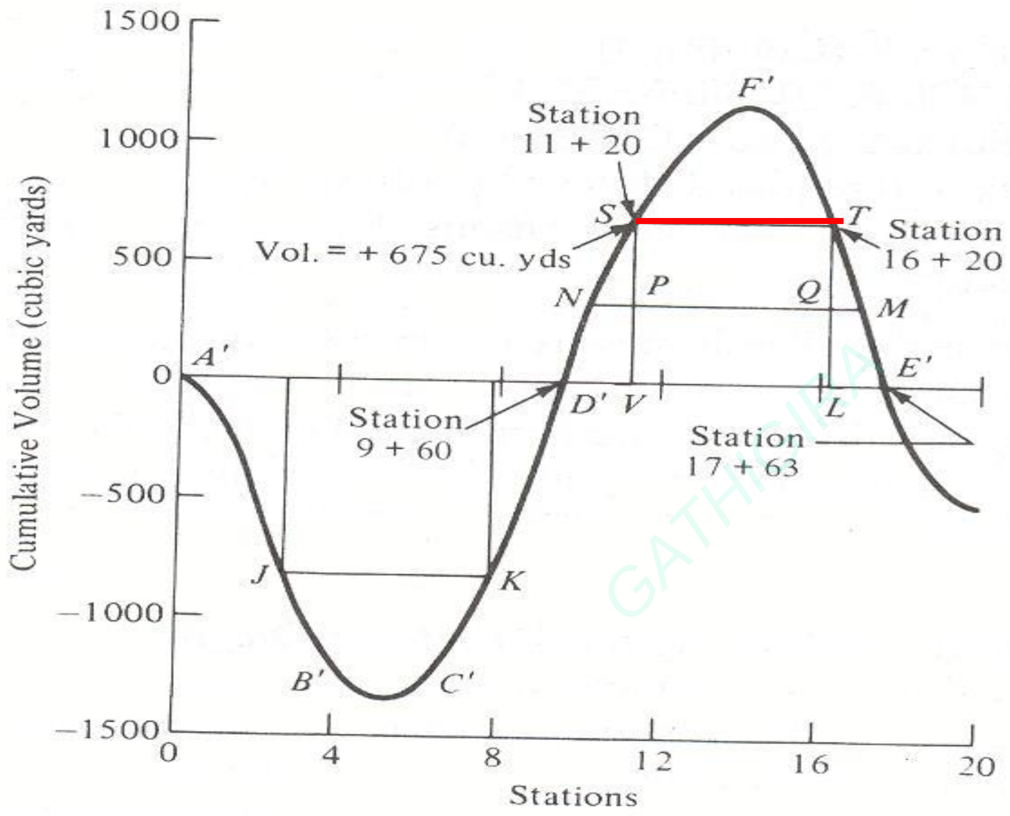
A′ and D′

D′ and E′

N and M

Etc.

note: a horizontal line defines locations where net accumulation between these two balance points is zero



Mass Haul Diagram for Computation Shown in Table 15.1

Locations of balanced cut and fill *JK* and *ST*

ST is 5 stations long

[16 + 20] – [11 + 20]

Special Terms

- Free haul distance (FHD)- distance earth is moved without additional compensation
- Limit of Profitable Haul (LPH) - distance beyond which it is more economical to borrow or waste than to haul from the project
- Overhaul – volume of material (Y) moved X Stations beyond Freehaul, measured in sta-yd^3 or sta-m^3
- Borrow – material purchased outside of project
- Waste – excavated material not used in project

Mass Diagram Development

1) Place FHD and LPH distances in all large loops
2) Place other Balance lines to minimize cost of movement
   - Theoretical; contractor may move dirt differently
3) Calculate borrow, waste, and overhaul in all loops
4) Identify stations where each of the above occur

Mass Diagram Example

- FHD = 200 m
- LPH = 725 m

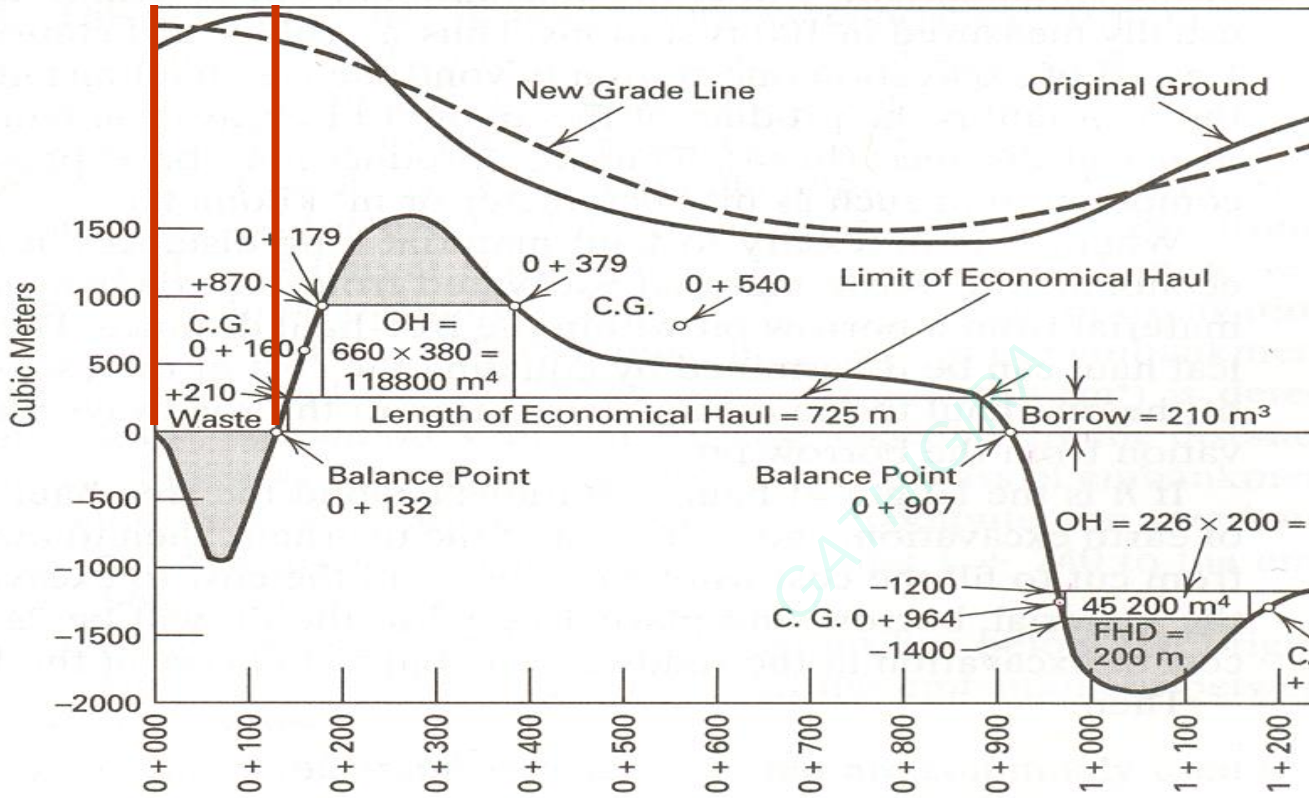


FIGURE 13-8 Example mass diagram. (NOTE: The numbers used he are for illustrative purposes only and may not be representative o values used in engineering practice.)

Between Stations 0 + 00 and 0 + 132, cut and fill equal each other, distance is less than FHD of 200 m

Note: definitely NOT to scale!

Source: Wright, 1996

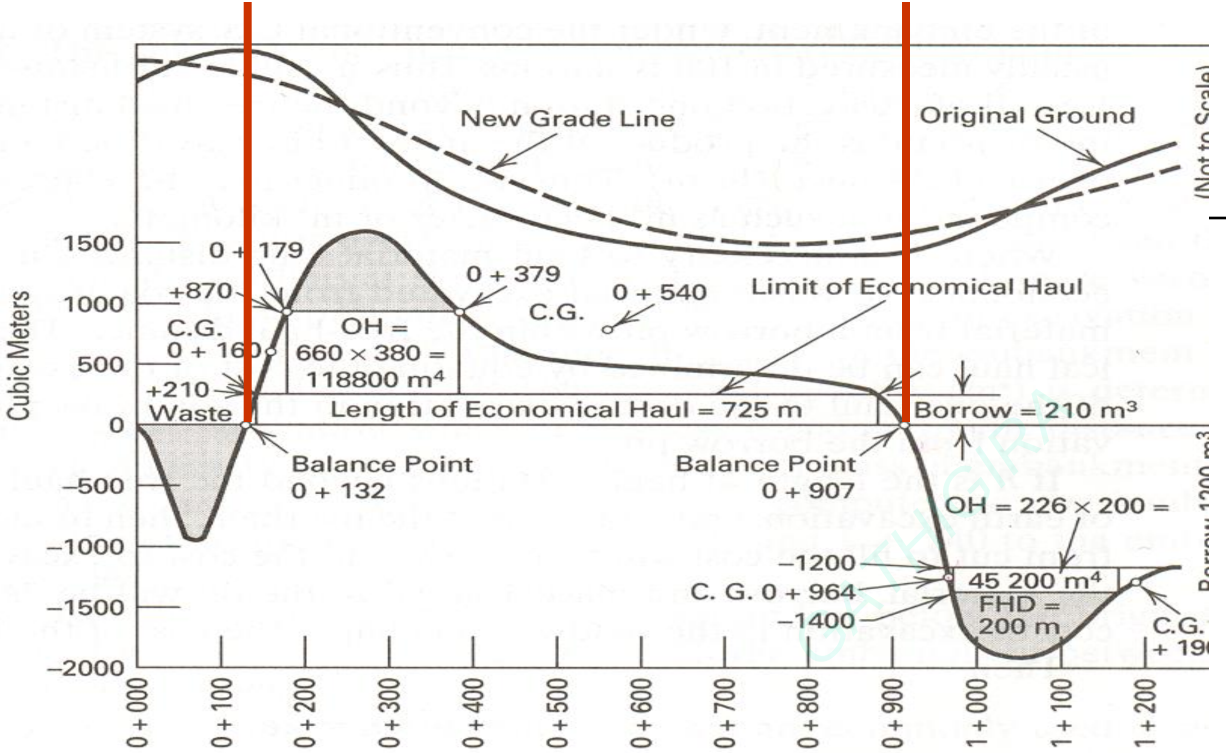


FIGURE 13-8 Example mass diagram. (NOTE: The numbers used here are for illustrative purposes only and may not be representative of values used in engineering practice.)

Between Stations 0 + 132 and 0 + 907, cut and fill equal each other, but distance is greater than either FHD of 200 m or LPH of 725 m

Distance = [0 + 907] – [0 + 132] = 775 m

Source: Wright, 1996

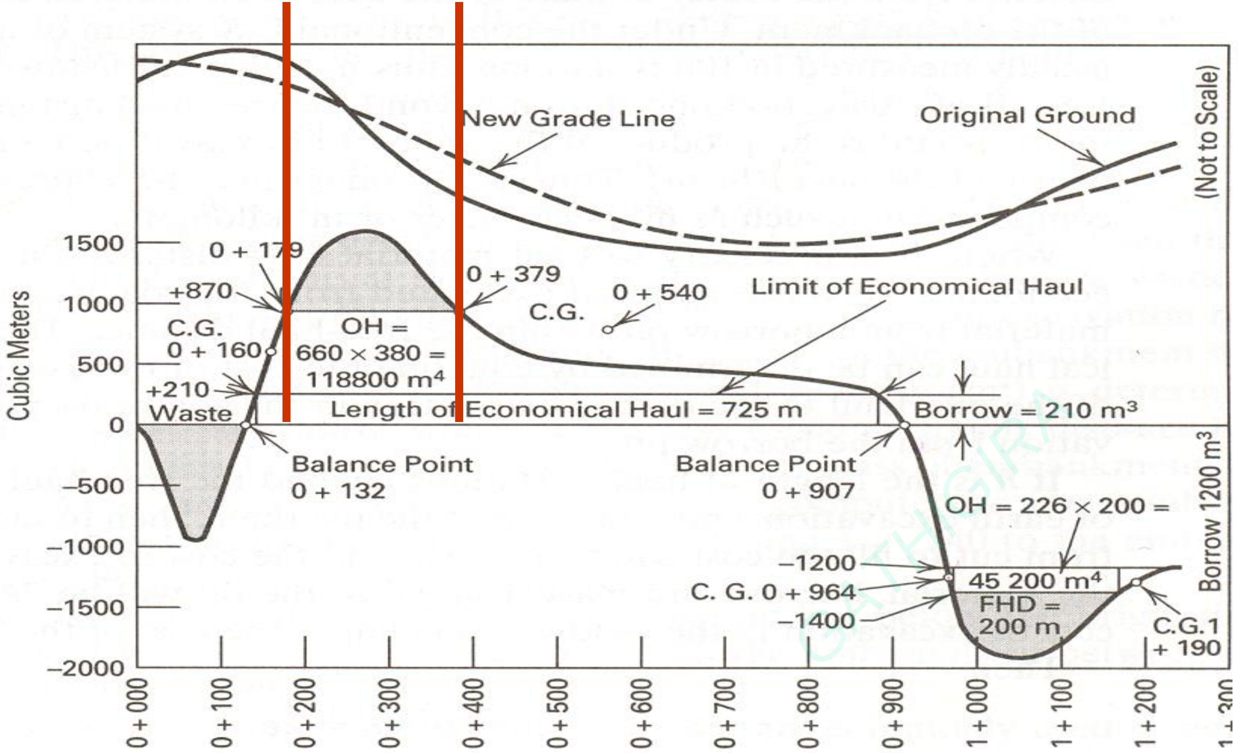


FIGURE 13-8 Example mass diagram. (NOTE: The numbers used here are for illustrative purposes only and may not be representative of values used in engineering practice.)

Between Stations 0 + 179 and 0 + 379, cut and fill equal each other, distance = FHD of 200 m

Treated as freehaul

Source: Wright, 1996

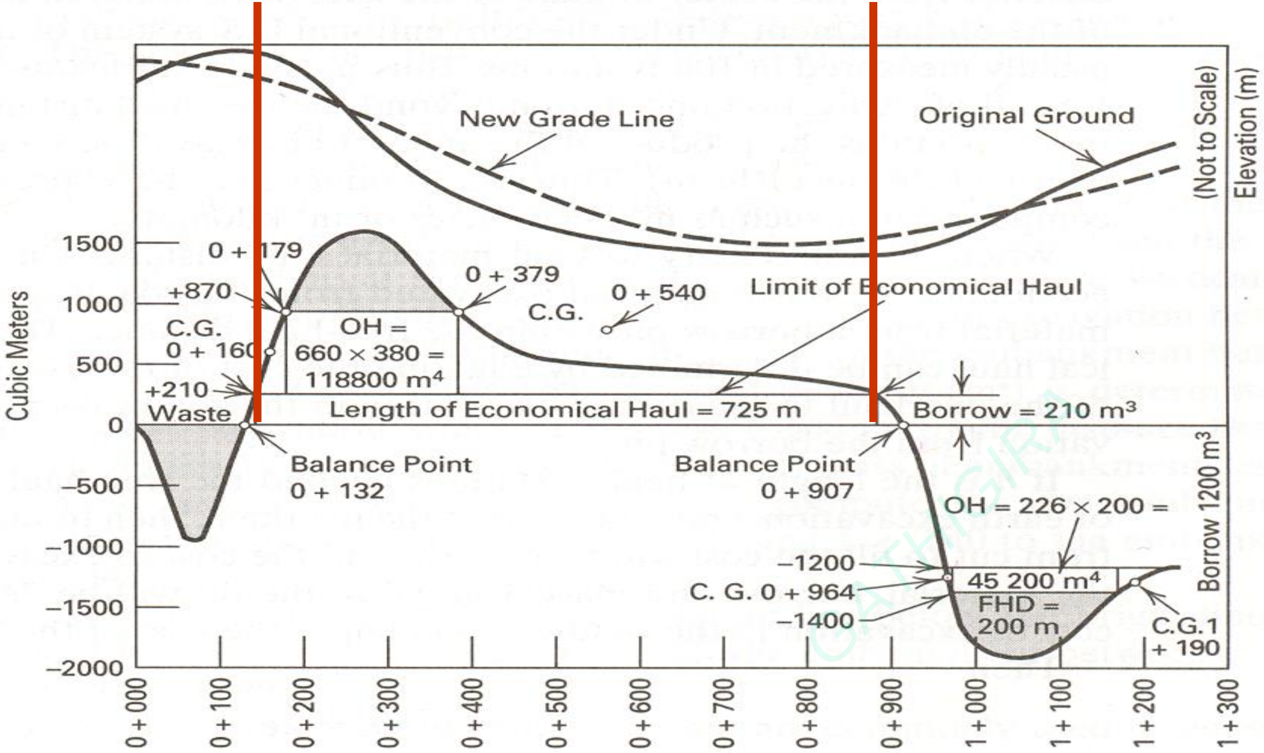
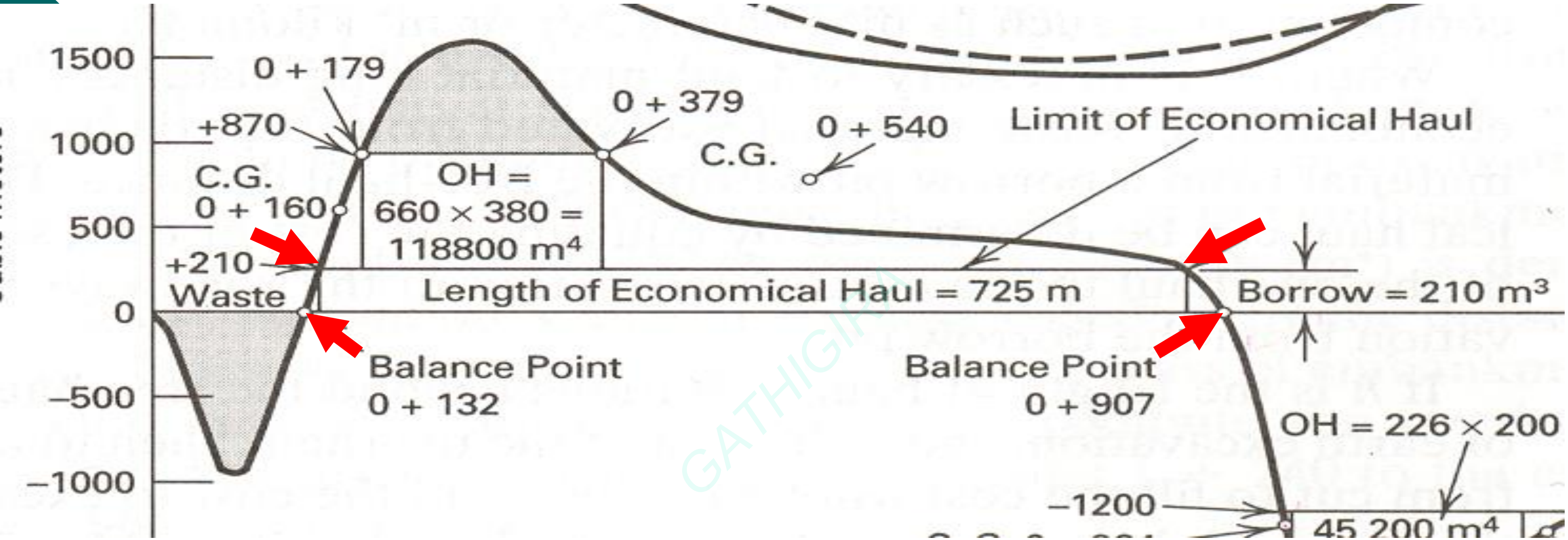


FIGURE 13-8 Example mass diagram. (NOTE: The numbers used here are for illustrative purposes only and may not be representative of values used in engineering practice.)

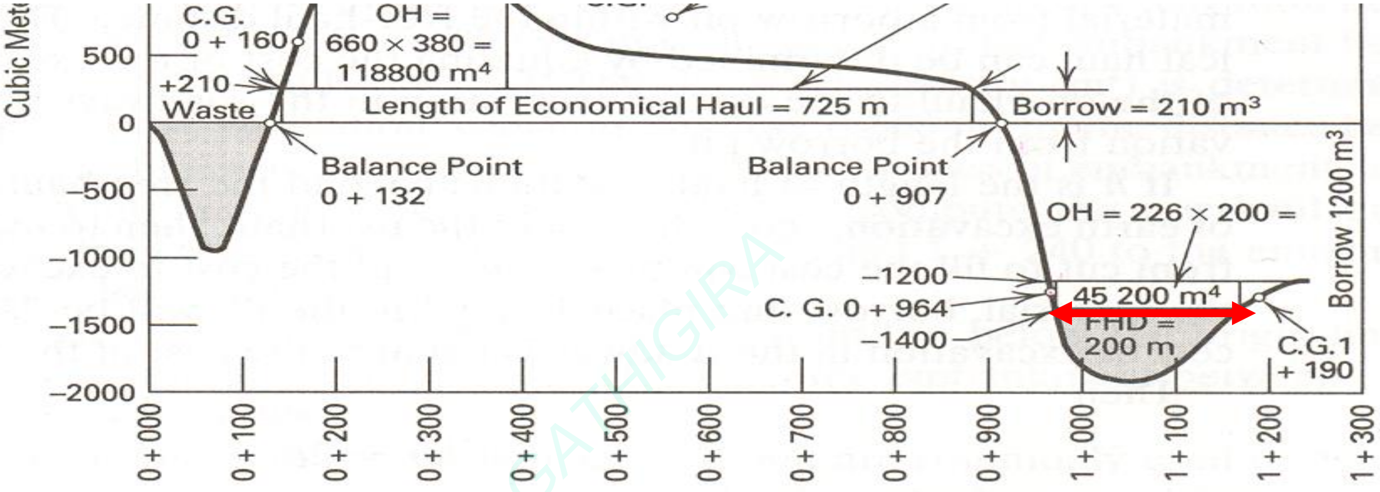
- Between Stations 0 + 142 and 0 + 867, cut and fill equal each other, distance = LPH of 725 m

Source: Wright, 1996



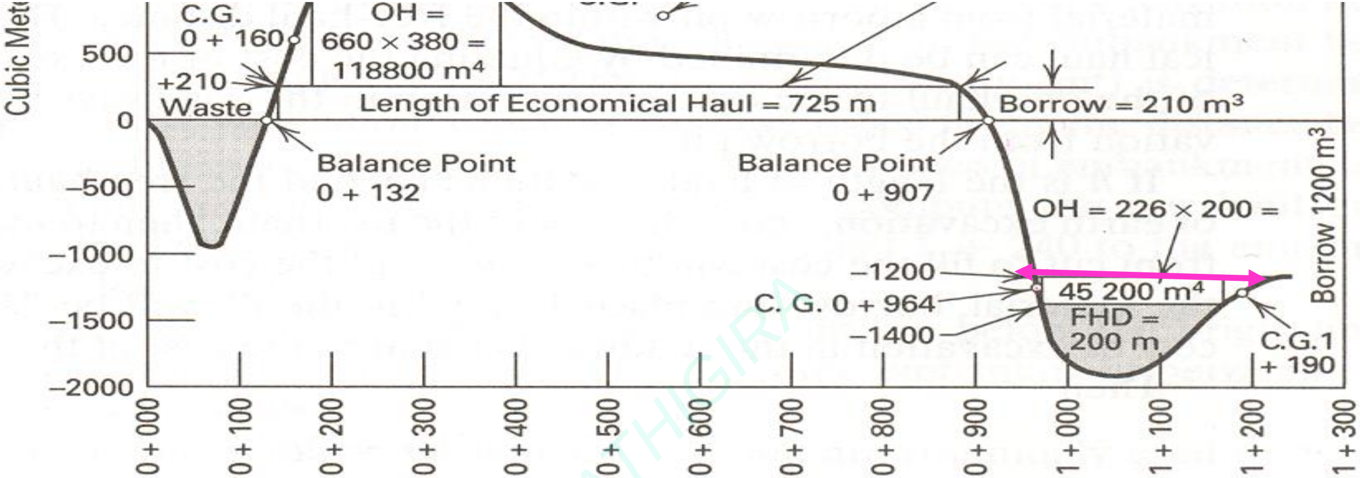
Material between Stations 0 + 132 and 0 + 42 becomes waste and material between stations 0 + 867 and 0 +907 becomes borrow

Source: Wright, 1996



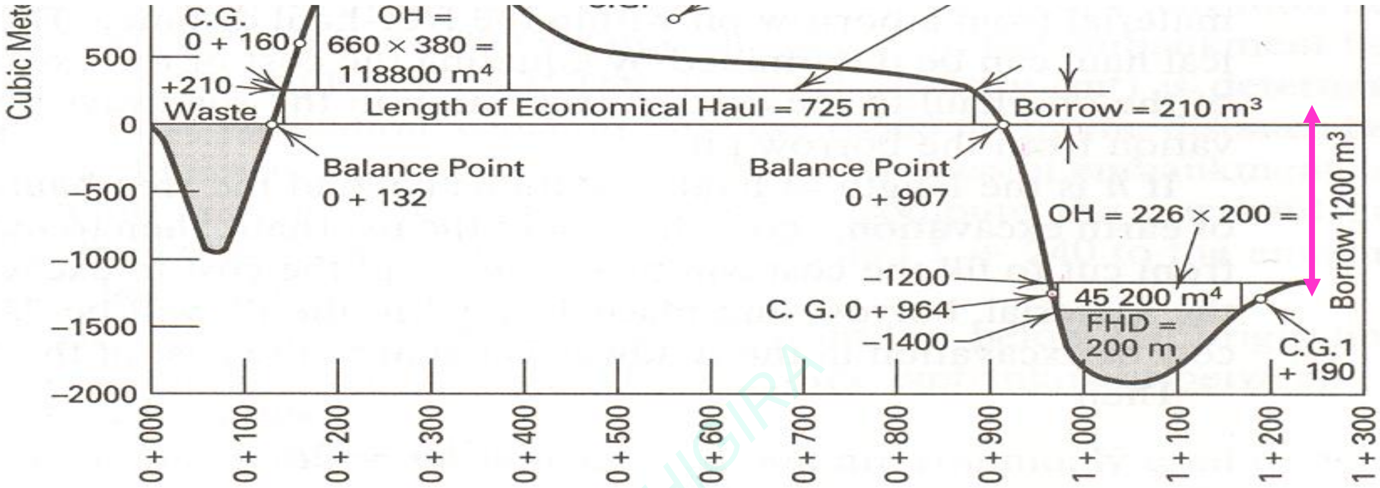
Between Stations 0 + 970 and 1 + 170, cut and fill equal each other, distance = FHD of 200 m

Source: Wright, 1996



Between Stations 0 + 960 and 1 + 250, cut and fill equal each other, distance is less than LPH of 725 m

Source: Wright, 1996



Project ends at Station 1 + 250, an additional 1200 m^3 of borrow is required

Source: Wright, 1996

Volume Errors

- Use of Average End Area technique leads to volume errors when cross-sections taper between cut and fill sections. (prisms)

- Consider Prismoidal formula

Prismoidal Formula

$Volume = (A_1 + 4A_m + A_2)/6 * L$

Where A_1 and A_2 are end areas at ends of section

A_m = cross sectional area in middle of section, and

L = length from A_1 to A_2

A_m is based on linear measurements at the middle

Consider cone as a prism

- Radius = R, height = H

- End Area 1 = πR^2
- End Area 2 = 0
- Radius at midpoint = R/2
- Volume $=((\pi R^2+4\pi(R/2)^2+0)/6) * H$
- $= (\pi R^2/3) * H$

Compare to “known” equation

- Had the average end area been used the volume would have been
- $V = ((\pi R^2) + 0)/2 * L$ (or H)

- Which Value is correct?

Class application

- Try the prismoidal formula to estimate the volume of a sphere with a radius of zero at each end of the section length, and a Radius R in the middle.

- How does that formula compare to the "known" equation for volume?
- What would the Average End area estimate be?

THANKS

- MORE ON THE LECTURES ON
- YOUTUBE PAGE
- IWORDS SHOTS EMPIRE

- SURVEY ENGINEERING
- BY GATHIGIRA STEPHEN
- MATILI TECHNICAL

REFERENCES

- FUNDAMENTAL OF SURVEYING
- SURVEY HANDBOOK
- SURVEYING MADE EASY NOTES
- SURVEYING BANNISTER
- MY NOTES

Disclaimer

- This document does not claim any originality and cannot be used as a substitute for prescribed textbooks. The information presented here is merely a collection by the author for their respective teaching assignments. We would like to acknowledge various sources like freely available materials from internet from which the lecture note was prepared. The ownership of the information lies with the respective authors or institutions. Further, this document is not intended to be used for commercial purpose and the author is not accountable for any issues, legal or otherwise, arising out of use of this document. The author makes no representations or warranties with respect to the accuracy or completeness of the contents of this document and specifically disclaim any implied warranties of merchantability or fitness for a particular purpose.

SUBSCRIBE ON MY CHANNEL
IWORDS SHOTS
EMPIRE

IWORDS
ACADEMY

School for the smarties